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Study nonlinear dynamics of stratospheric ozone concentration at Pakistan Terrestrial region

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ABSTRACT

This study investigates the nonlinear dynamics of the stratospheric ozone layer at Pakistan atmospheric region. Ozone considered now the most important issue in the world because of its diverse effects on earth biosphere, including human health, ecosystem, marine life, agriculture yield and climate change. Therefore, this paper deals with total monthly time series data of stratospheric ozone over the Pakistan atmospheric region from 1970 to 2013. Two approaches, basic statistical analysis and Fractal dimension (D) have adapted to study the nature of nonlinear dynamics of stratospheric ozone level. Results obtained from this research have shown that the Hurst exponent values of both methods of fractal dimension revealed an anti-persistent behavior (negatively correlated), i.e. decreasing trend for all lags and Rescaled range analysis is more appropriate as compared to Detrended fluctuation analysis. For seasonal time series all month follows an anti-persistent behavior except in the month of November which shown persistence behavior i.e. time series is an independent and increasing trend. The normality test statistics also confirmed the nonlinear behavior of ozone and the rejection of hypothesis indicates the strong evidence of the complexity of data. This study will be useful to the researchers working in the same field in the future to verify the complex nature of stratospheric ozone.

1. Introduction

Although, Ozone now considered an important element in the earth biosphere because of its role to protect life of human being on the earth's surface from harmful ultraviolet (UV) solar radiation (Jan et al., 2014). Moreover, ozone may have indispensable relations to weather and responsible for climate fluctuations (Whitten and Prasad, 1985; Werner, 2008). Over the past decade's research have been increased a widespread awareness of the importance of ozone because of anthropogenic actions in the enhancement of surface UV radiation, which can increase hazardous moments for human and biosphere (Varotsos, 2000). Since, ozone is a powerful absorber of UV radiations, its existence in the upper atmosphere is consequence of local heating there and the formation of the stratosphere (Whitten and Prasad, 1985). The attentions of stratospheric ozone in terrestrial life being very imperious for sustaining life near the earth environment by absorbing havoc UV radiation within the stratosphere (Matasović et al., 2014). The dynamic of ozone is a universal issue and it became a serious hazard to human health, adverse impact on animals, lower productivity of vegetation, building damage, lack of agriculture yield, economical deficiency and ecological system (Weng et al., 2008; Varotsos, 2000). Ozone fluctuations are caused by changing at the rate of photochemical reactions and transport connected with meteorological conditions. Volcanic eruptions are inaccessible huge events that can perturb the dynamics of ozone in the stratosphere (Poulin and Evans, 1994). These emissions consist of huge masses of SO_4, H_2O and HCl, which are able to change the chemistry of ozone containing compounds and enhance aerosol on stratosphere as a result ozone variation occurs. Atmospheric chemical pollutants, $CFCl_3$, CF_2Cl_2, N_2O, and CH_4 direct to photochemical interactions that influence ozone level. Increase in N_2O and CH_2ClF have significant effects on surface warming due to the direct radiative interaction and their dissociation in the stratosphere enhanced the production of NO_x and ClO_x reactive elements that can catalytically eradicate thickness of ozone by cooling or warming stratosphere (Whitten and Prasad, 1985).

Since, ozone formation is an intricate and nonlinear dynamical process relating nitrogen and turbulent organic compounds present in tropospheric region. Because of high insulation and temperature persistency in intense meteorological conditions, ozone concentration becomes

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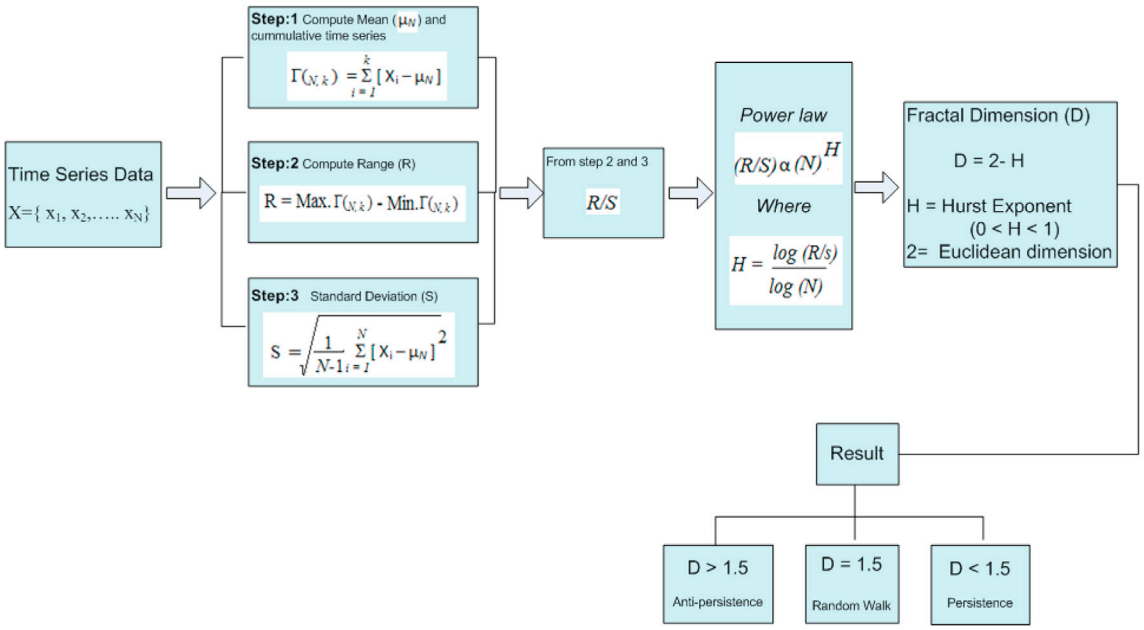

Fig. 1. Step by step process of Hurst exponent and Fractal dimension on perspective of Rescaled ranged analysis.

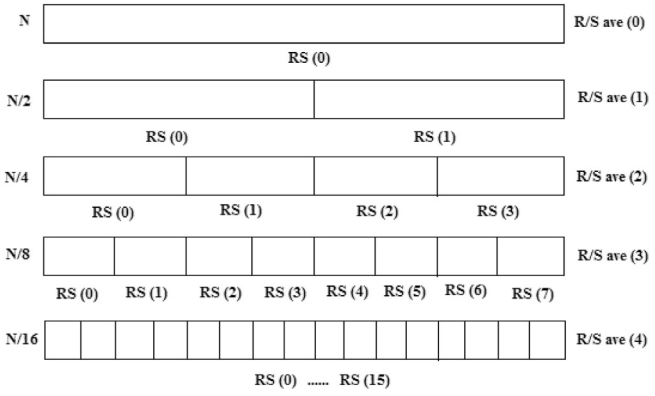

Fig. 2. Estimation of rescaled range analysis method to computing Hurst Exponent (H).

significantly higher. A large scale of ozone concentration takes place in a high anticyclone (high pressure) regions (Banja and Porja, 2008). The meteorological parameters (i.e. clouds, wind direction, wind speed, temperature and humidity) play a significant role in the variability of ozone concentration. Fluctuations in meteorological conditions can have a strong affect on ozone behavior, formation and transport from the stratosphere. Clouds affect intensity of UV radiations and maximum temperature (Whitten and Prasad, 1985).

Weather patterns are also important to ozone formation because of the long time of sunshine and highest temperature during summer season (Kgabi and Sehloho, 2012). The temperature and wind speed both shown a strong positive correlation with Ozone while relative humidity correlated negatively (Li et al., 2007). Variability of climate change in earth surface can influence the behavior of the ozone layer, as ozone is biased by alteration in the meteorological systems and fluctuation in the atmospheric composition that can effect from climate change. The most important problem is that the stratosphere maybe cool due to climate change, therefore maintaining of climate conditions for a long time that enhance chlorine (Cl_2) in the upper troposphere as a result, ozone reduction increased in the tropopause, mainly in the polar areas (Markovic and Markovic, 2005). The development of ozone takes place by the interaction of O_2 with solar UV radiation (Jilani and Zai, 2011) and this dynamic of ozone (Sych et al., 2005) over Pakistan region has been studied by several researchers (Zai and Khan, 2006; Zai et al., 2009; Zahid and Ghulam, 2010). The ozone is considered as one of the important atmospheric elements to understand system of chemical and dynamical atmospheric phenomena (Werner, 2008).

The Fractal approach applied to a wide range of natural events, that is, it shows 'self-similarity' which stress the broad-spectrum of the object is usually repetitive at randomly smaller and smaller levels (Kale and Butar, 2011; Sachithananthem, 2012). The Fractal Dimension (D) exhibits strong nature of complexity and nonlinear dynamics of natural complex systems (Zai, 1989). A fractal nature with a high convolution is much complex or 'rough' on contrast to a lower dimension and fills much space (Hassan et al., 2014). The Fractal analysis applied in advance scientific discipline of time series data like Space Weather (Sunspot or flares), Climate Change, Biological Science, Quality Control System, Population Dynamics, Astronomy, Rbeology, Economics, Industry and Engineering (Kaye, 1993). We adopt two most recent advance techniques of the fractal dimension to estimate the Hurst exponent (H) by Rescaled range analysis (R/S) and Detrended fluctuation analysis (Krištoufek, 2010).

Mandelbrot and Van Ness (1968) were the first who has applied Hurst exponent approach for fractal dimension and this methodology has practical application in time series data. Kassem (2014) applied it to investigate persistency in time series data of hourly and monthly surface ozone level in Egypt on the perspective of Rescaled range analysis. Sachithananthem (2012) predicted behavior of ground O_3 level concentration as a random series in south Chennai based on fractal and Hurst exponent. SureshKannan and Steephen (2012) studied the concept of fractal analysis, Hurst exponent on time series data of Ozone. He observed that the total column of O_3 over the tropical region shown persistence behavior. Weng et al. (2008) study based on tropical ozone dynamics in Taiwan for analysis the nonlinear dynamics of ground ozone level on perspective of the rescaled range analysis (R/S). Their finding highlights a strong persistent behavior in tropical ozone level.

The Detrended fluctuation analysis (DFA) was another approach which is an extension of R/S analysis to observe the fractal character of time series data. It was applied to examine the long-range power-law correlations properties (Selvaraj, and Uma, 2012). Many researchers studied DFA in different areas like physics, engineering, and finance,

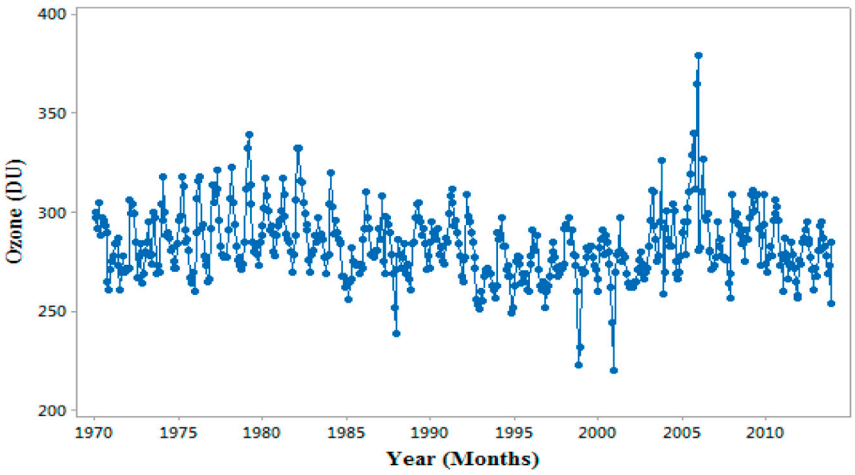

Fig. 3. Plot of the monthly average time series pattern of stratospheric ozone at Pakistan region from 1970-2013.

Table 1
Summary basic statistics monthly and seasonal ozone (DU) data for the study period for 1970-2013.

Statistic	Spring	Summer	Autumn	Winter	Monthly
Number of Count	44	44	44	44	528
Minimum	262.34	270.00	262.67	266.56	283.43
Maximum	328.34	310.34	327.00	309.00	220.00
Median	291.00	286.67	277.34	286.39	379.00
Mean	293.21	287.11	277.47	285.93	283.43
St. Deviation	15.52	9.50	10.78	9.66	17.25
Skewness	0.08	0.32	2.57	0.10	0.73
Kurtosis	-0.26	-0.16	9.26	-0.18	3.02
KS statistics	0.08	0.06	0.18	0.05	0.07

medical as well as meteorology to investigate scaling exponent (Hurst exponent). Zhang et al. (2011) and Matsoukas et al. (2000) applied different Detrending approaches to observe fluctuation in hydrological data. It was also applied around 700 heat rate variability (HRV) studies since 1995 when DFA was introduced first time to evaluate the correlation properties of signals (Perakakis, 2009). Wu et al., 2007 performed detrending analysis on global surface air temperature anomaly (GSTA) to determine variability of trend line as well as some interesting nonlinear characteristics of the data. .

To observe the intrinsic nature of atmospheric ozone in Pakistan, this research inspects complex nature of stratospheric ozone over the Pakistan atmospheric region carefully by conducting fractal dimension associated with Hurst exponent using R/S analysis and Detrended fluctuation method.

This study distributed into different sections. In section: II, we expressed data and all techniques in briefly. In section: III, we show results with detail discussions and in section: IV, conclude the results.

2. Data and method of analysis

The monthly time series data of O_3 covering the period from 1970 to 2013 for the Pakistan atmospheric region were obtained from World Ozone and Ultraviolet Radiation Data Centre (WOUDC) and Pakistan Meteorological Department (PMD), Quetta Centre. The data were explored using statistical software Minitab 17, the E-View and package 3.2.5 distributed by R software.

The important methodology carried out in this communication may be classified into following parts.

2.1. Basic statistical analysis

The application of statistical analysis commonly used in every field, including space science, climatology and meteorology. Basic statistical analysis is one of a dominant approach in atmospheric science to the observed behavior of time series data (Zai et al., 2013). It determined descriptive statistics and also conduct normality test for time series data using different test like Anderson-Darling (AD), Kolmogorov-Smirnov (KS) and Shapiro-Wilk (SW). For this purpose, we have selected the AD and KS test statistic that is defined mathematically as:

$$A^2 = -N - S$$

and,

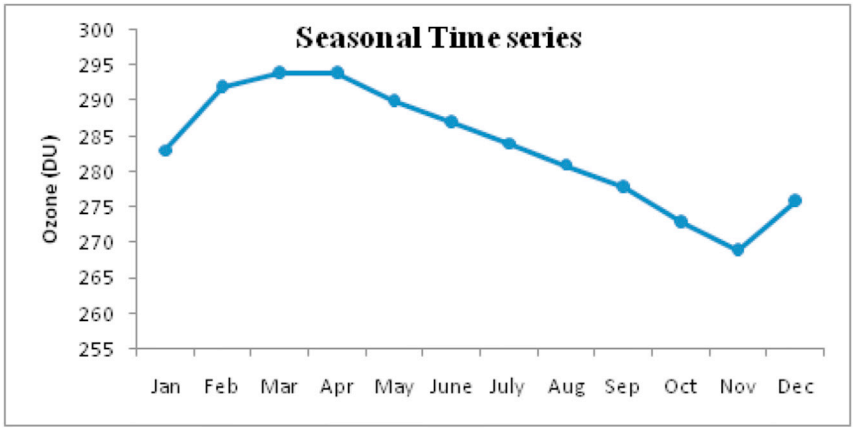

Fig. 4. Plot of the seasonal average time series pattern of stratospheric ozone at Pakistan region from 1970-2013.

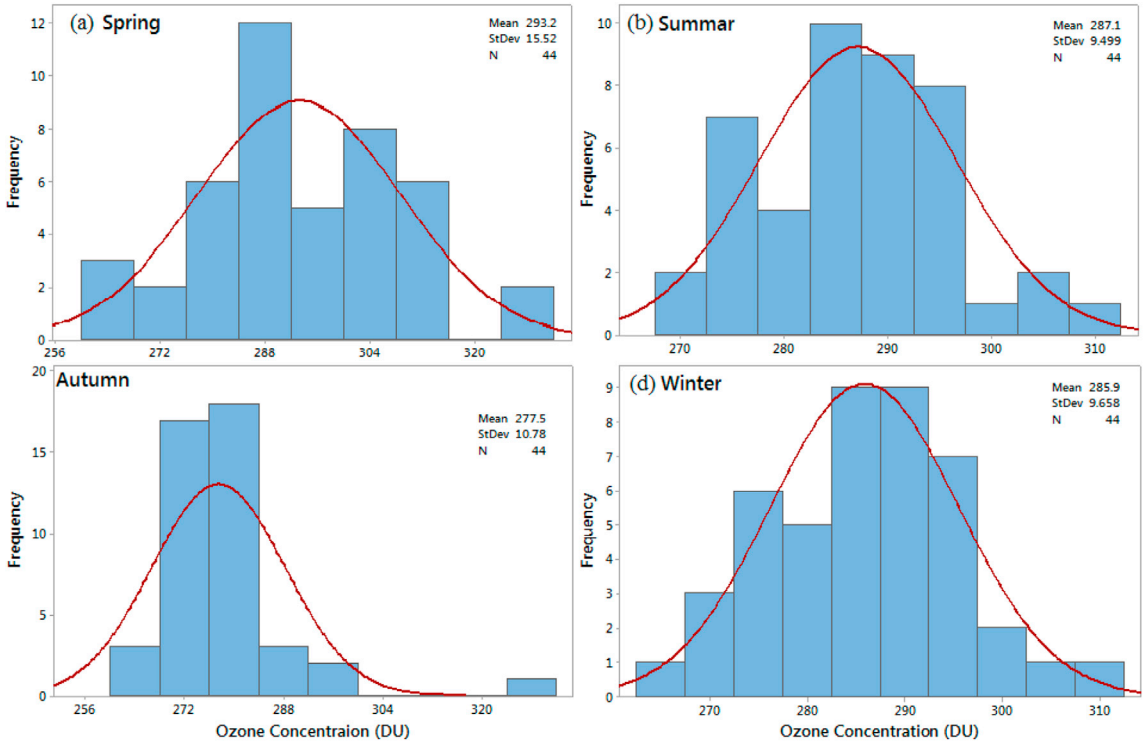

Fig. 5. Average seasonal plots of histograms of stratospheric ozone concentration (DU) for (a) Spring, (b) Summer, (c) Autumn and (d) Winter for 1970-2013.

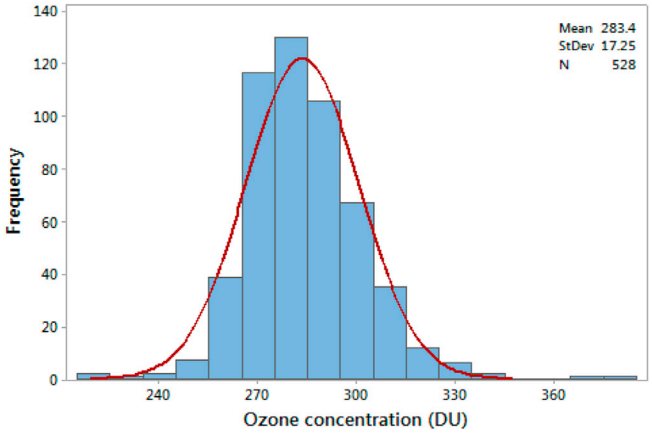

Fig. 6. Histogram plot of average monthly stratospheric ozone concentration (DU) from 1970-2013.

$$S = \sum_{i=1}^{N} \frac{(2i-1)}{N} [\ln F(x_i) + \ln(1 - F(x_{N+1-i}))] \quad (1)$$

Where, F is defined the cumulative distribution function (cdf) of the distribution being tested and x_i are ordered. This test is one sided and we reject the null hypothesis if the value of A is larger than the critical value. Also, the value of prob. $> A^2$ is the p-value. If the p-value is less than the level of significance α then null hypothesis is rejected (Kale and Butar, 2011). Likewise, the KS statistic is presented mathematically as under:

$$D = \sum_{1 \le i \le N} \left| F(x_i) - \frac{n(i)}{N} \right| \quad (2)$$

Where, F is defined as the theoretical cdf of continuous distribution which being tested. The selection of hypothesis is as under, if the test statistic D is larger than the critical value obtained from the table then the null hypothesis is rejected. The value Prob. > D is the p-value. This emphasizes that lower the p-value from α then null hypothesis rejected and data are normally distributed (Zai et al., 2013).

For better understanding the statistics of ozone variability, the correlations between data at different points in time lagged by one or more periods, an autocorrelation and a partial autocorrelation analysis has been conducted (Subyani and Hajjar, 2016). Mathematically, it can be represented as:

$$ACF(k) = \frac{\sum_{i=1}^{n-k} (x_i - \mu_x)(x_{i+k} - \mu_x)}{\sum_{i=1}^{N} (x_i - \mu_x)^2} \quad (3)$$

Where, x_i from 1……N is the time series data and μ_x for mean of data, k for lag index in months. The statistical software's were used in this research are MINITAB version 17 and EViews version 5.

2.2. Hurst exponent (H)

Hurst introduced in 1965, the well known statistical methodology rescaled range analysis (R/S) to compute time series data of natural event (Kale and Butar, 2011). This approach estimates the cumulative series of time series data which deviated from its mean (López-Montes et al., 2012). Several researchers (Kassem, 2014; SureshKannan and Steephen, 2012; Kale and Butar, 2011; Krištoufek, 2010) used to compute Hurst exponent of time series data by application of the Rescaled range analysis and Detrended fluctuation analysis.

2.3. Resealed range analysis (R/S)

The complete mathematical procedure of Hurst exponent (H) on the perspective of the Rescaled range analysis performed with complete steps by following flowchart as shown in Fig. 1. The procedure was repeated for whole samples (N) over the time series data and dividing each sample interval by two and determining R/S for each sub samples as revealed in

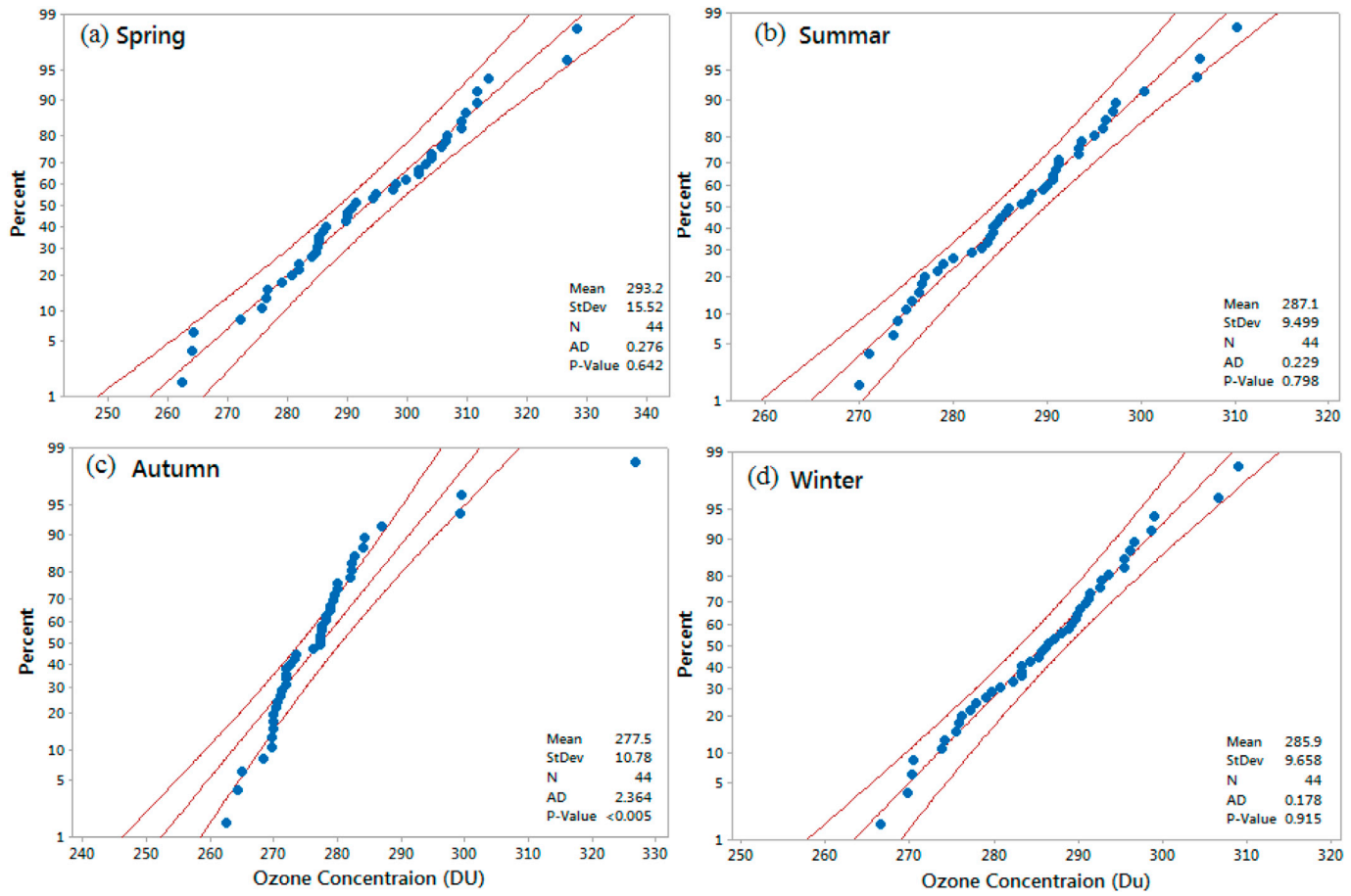

Fig. 7. Average seasonal plots of best fitted distribution normal probability density function (pdf) for stratospheric ozone (DU)(a) spring, (b) summer, (c) autumn and (d) winter for 1970-2013.

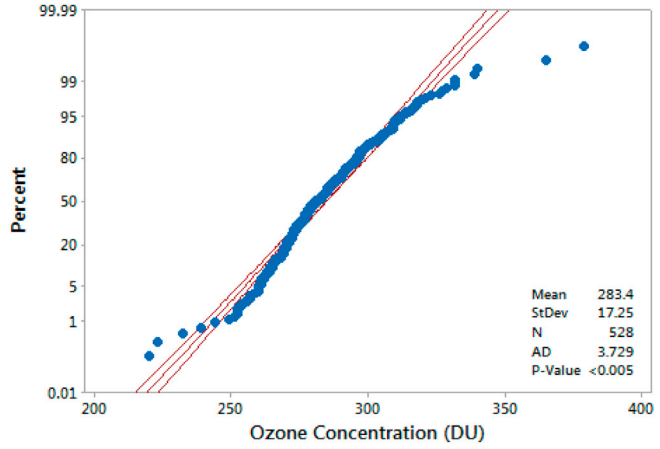

Fig. 8. Plot of best fitted distribution normal probability density function (pdf) of monthly stratospheric ozone concentration (DU) from1970-2013.

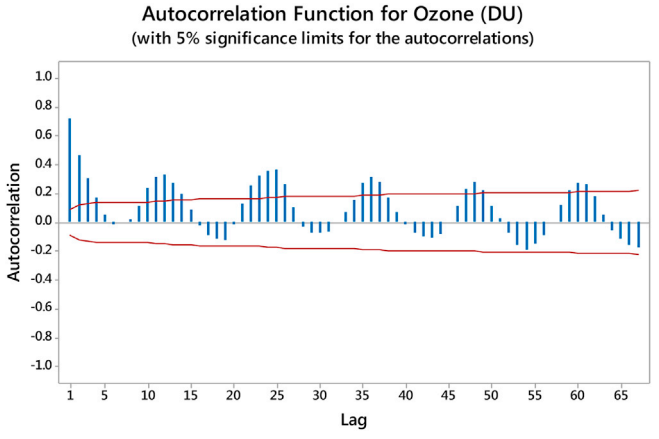

Fig. 9. Plot of autocorrelation function of monthly ozone concentration time series data at Pakistan atmospheric region from 1970-2013.

Fig. 2.

2.4. Detrended fluctuation analysis (DFA)

DFA applied to compute correlation properties of time series data, obtaining from financial (Krištoufek, 2010), biological, physical as well as social structure (Perakakis, 2009). One of application of DFA was that it is not necessary to check the stationarity of time series data. In order to determine DFA, consider the time series $\{O_i\}$, i = , 2, 3 … N, where N is size of ozone data series. The DFA process determined by following the steps as:

Step: 1 First determine the time series y (n),

$$y(n) = \sum_{i=1}^{n} (O_i - \overline{O})$$

Where, $\overline{O}$ represent mean of time series data.

Step: 2. In this step, divide the integrated series into boxes of equal

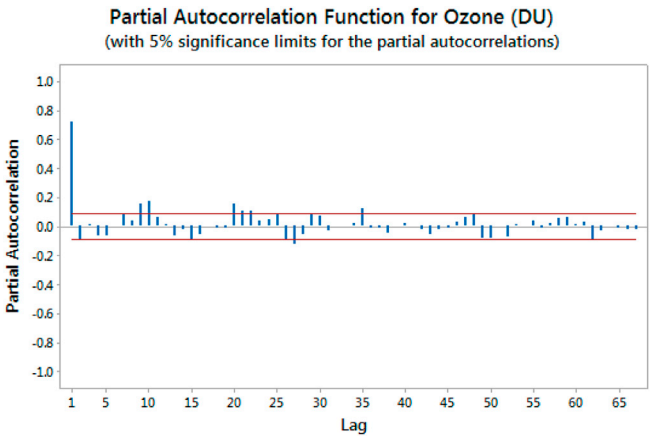
Partial Autocorrelation Function for Ozone (DU)
(with 5% significance limits for the partial autocorrelations)
Partial Autocorrelation
Lag

Fig. 10. Plot of partial autocorrelation function of monthly ozone concentration time series data at Pakistan atmospheric region from 1970-2013.

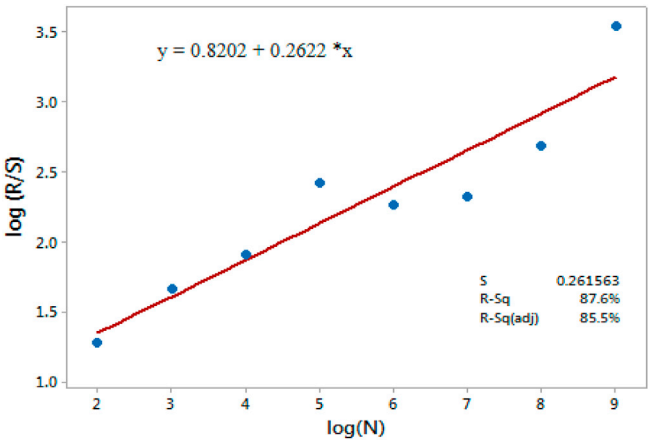

Fig. 11. Plot of R/S analysis, the least square procedure for ozone concentration for Pakistan atmospheric where the slope of fitted line estimates the Hurst Exponent for the year 1970-2013.

Table 2
Summary of the ADF test statistics for different significance.

		t-Statistic	Prob.[a]
Augmented Dickey-Fuller test statistic		-9.272533	0.0000
Test critical values:	1% level	-3.442530	
	5% level	-2.866805	
	10% level	-2.569635	

[a] One-sided p-values.

Table 3
The detail procedure of rescaled range analysis for ozone concentration over Pakistan atmospheric region to estimate Hurst for year 1970-2010.

Level	Data size (N)	R/S	$\log_2(N)$	$\log_2\left(\frac{R}{S}\right)$
1	512	8.58372	9	3.5440
2	256	6.17304	8	2.6898
3	128	4.42869	7	2.3195
4	64	4.21956	6	2.2609
5	32	3.48272	5	2.4250
6	16	3.31753	4	1.9098
7	8	3.33333	3	1.6661
8	4	2.34314	2	1.2814

size s, then designed fit of the local trend by least square procedure for each segment.

Step: 3 After that, each segment detrended by subtracting a least squares linear trend $\hat{y}(n)$ with Y(n) i.e.

$$y(n) - \hat{y}(n) \tag{4}$$

Step: 4 Similarly, intended the root-mean-squares (RMS) fluctuation of the original time series and Detrended time series as:

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[y(n) - \hat{y}(n)]^2} \tag{5}$$

Step:5 Finally, determine the scaling exponent (α) by the relation:

$$F(n) \sim n^{\alpha} \tag{6}$$

The procedure is repeated to all segments over the range of box sizes to present a relationship between F(n) and n is obtained (Zhang et al., 2011; Selvaraj, and Uma, 2012). A linear association between log-log plot of F(n) verses 'n' gives the fractal estimation of the scaling exponent (also called Hurst exponent) 'α' is given by the gradient (slop). The value of 'α' is interpreted as follows, if the value of α is zero than there is no correlation, a scaling exponent of $\alpha > 0.5$ indicates the series is long range positive correlation and $\alpha < 0.5$ correspond an anti-correlation in original time series (Bashan et al., 2008; Perakakis, 2009; Krištoufek, 2010).

2.5. Fractal dimension (D)

Since, the Fractal dimension emphasized the roughness of time series data (Rehman and Siddiqi, 2009) whereas the Hurst exponent estimates the smoothness (Kale and Butar, 2011). The fractal dimension (D) directly linked with Hurst exponent (H) according to the function and the relation can compute by:

$$D = 2 - H \tag{7}$$

Where, D stands for fractal dimension, H represent the Hurst exponent while 2 stand for Euclidean dimension. Both fractal dimension and H illustrated the dynamical behavior of the time series data (López-Montes et al., 2012). The H is limited to the range $0 \leq H \leq 1$ and that compares the nature of the time series data (Hassan et al., 2014). When the value of $H > 0.5$ ($D < 1.5$), we have a persistent time series (positive correlation), if the value of $H < 0.5$ ($D > 1.5$), the process is considered as anti-persistent (negative correlation) time series and if $H = 0.5$ ($D = 1.5$) indicates that the system is "random walk" i.e. the time series is independent and uncorrelated. Furthermore, as the value of H moves towards zero, it implies that the irregularity of the system rose up (Kassem, 2014; López-Montes et al., 2012; Kale and Butar, 2011).

3. Results and discussions

3.1. Analysis of basic statistics results

The plan behind this study is to pay attention on the nonlinear dynamics of Ozone level in the stratospheric region of Pakistan. In the first step we have carried out the basic statistical analysis to observe the linear dependency and non-stationarity of data and then eliminate its linear dependency by estimating AR(p) residuals before performing R/S analysis (Weng et al., 2008). For this purpose, first introduced brief description of monthly and seasonal stratospheric ozone variations and its dynamics using statistical normality test defined the atmospheric region from 1970 to 2013. Usually the normality test can classified into two groups, descriptive statistics and assumption possessed techniques. Skewness and kurtosis coefficients are defined as descriptive statistics whereas other techniques consist of the normality tests such as AD, KS and SW tests. (Razali and Wah, 2011). In our case, we have chosen KS and AD test to see the normality of ozone time series data.

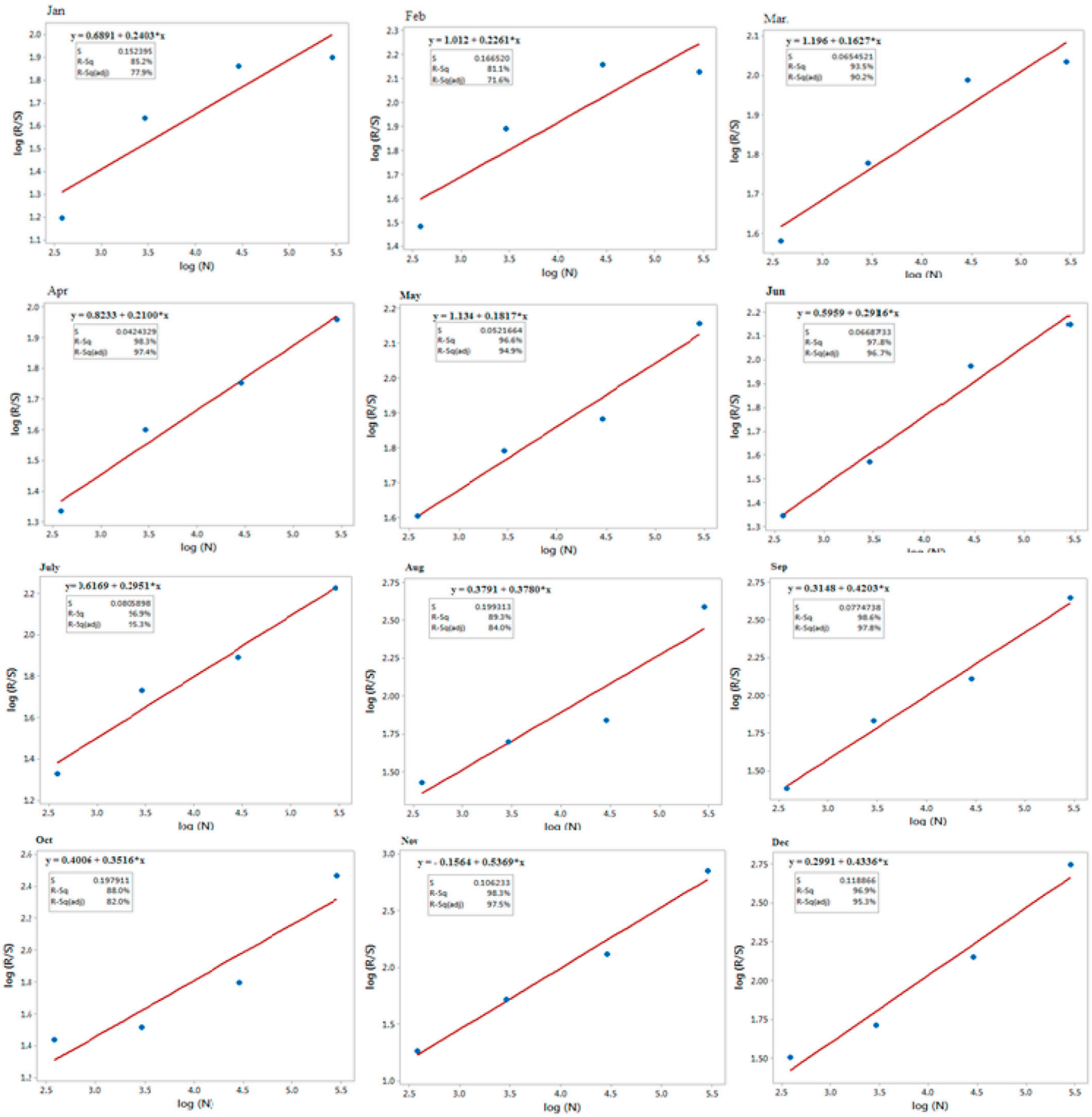

Fig. 12. Plot of R/S analysis, the least square procedure for seasonal pattern of stratospheric ozone concentration at Pakistan atmospheric region, where the slope of fitted line indicates Hurst Exponent for the year 1970-2013.

The monthly time variation of stratospheric ozone in the Pakistan atmospheric region has revealed in Fig. 3. It shows a clear variation of monthly average ozone (DU) during the time interval. The maximum ozone concentration recorded 379 DU and minimum 220 DU as indicated in summary Table 1. The seasonal pattern of stratospheric ozone is shown in Fig. 4 that revealed the seasonal peak of ozone concentration from February to April. This may be outcome of more production of ozone due to photochemical reactions in stratosphere region or meteorological parameters. The average maximum concentration recorded in March and minimum in November. The ozone production is usually to be higher for the duration of high sunshine hours, at night time, ozone depletion occurs due to its reaction with NO (Sharma et al., 2016). Since, skewness is defined as an indicator that applied in distribution analysis as a sign of irregularity and deviation from normal distribution (Kassem, 2014). From Table 1, the positive skewness indicates a sign of right skewed allocation of ozone concentration. This result claimed that mainly of values is determined on left and extreme values of the right of the mean. Kurtosis illustrates the vertical peakedness or smoothness of a distribution as compared to normal distribution. In our case, kurtosis are seasonally negative accept autumn and entire period. The negative kurtosis declared a quite smooth distribution, large wide peak as shown in frequency histograms Fig. 5 (a), (b) and (d). The positive kurtosis here indicated a peaked distribution as shown in Fig. 5 (c) for seasonal and for entire total months in Fig. 6, which is one of the signs of fractal brown

Table 4
Summary of Fractal dimension and Hurst Exponent of seasonal ozone concentration at Pakistan terrestrial region from 1970-2013.

Months	D	H	Nature	Correlation
Jan	1.7597	0.2403	Anti-persistent	Negative
Feb	1.7739	0.2261	Anti-persistent	Negative
Mar	1.8371	0.1627	Anti-persistent	Negative
Apr	1.7900	0.2100	Anti-persistent	Negative
May	1.8183	0.1817	Anti-persistent	Negative
Jun	1.7084	0.2916	Anti-persistent	Negative
July	1.7049	0.2951	Anti-persistent	Negative
Aug	1.6220	0.3780	Anti-persistent	Negative
Sep	1.5797	0.4203	Anti-persistent	Negative
Oct	1.6484	0.3516	Anti-persistent	Negative
Nov	1.4631	0.5369	Persistence	Positive
Dec	1.5664	0.4336	Anti-persistent	Negative

nature (Weng et al., 2008), these results representing more dynamic and intermittency in ozone level over Pakistan air space. The variance is also quite irregular and large.

The technique of the normality test was adopted to identify linear dependence in time series data of ozone concentration at Pakistan atmospheric region. Table 1 also described the summary of normal distribution test of Kolmogorov-Smirnov (KS) on the response of KS statistics and p-value for goodness of fit. The best fitted distribution in AD has been the choice as largest p-value and lowest AD statistics (Sharma et al., 2016). In addition, the overall time series data of ozone somewhat closed to normal as in Fig. 6 but failed to fallow the normal distribution test using the Kolmogorov-Smiirnov (KS) and AD statistics proposed by equations (1) and (2). Similarly, for seasonal series data of ozone concentration Spring, Summer and Winter are quite closed to normal distribution under supposed distribution test while Autumn quite away from normal. The output of best fitted normal probability distribution function (pdf) of ozone concentration for Pakistan air space, seasonal as well as the whole months from 1970 to 2013 are shown in Figs. 7 and 8 respectively. In Fig. 8 the departure of series data from extreme, indicates that the data do not fallow normal distribution.

The autocorrelation defined as the self correlation between the objective data points (Alam et al., 2014). This technique determined as a function of the lag as in equation (3) related to ozone time series data are expressed in Fig. 9 and partial autocorrelation is shown in Fig. 10, this indicates that the linear robust long-range interdependence clearly present in time series data of ozone over Pakistan region. This is one of the particular behaviors of nonlinear dynamic systems. Therefore, further nonlinearity work has to be figured out for strong fractality using R/S analysis and Detrended fluctuation analysis. Before the choice of model, we have checked the level of stationary of ozone time series data that can be properly confirmed by applying unit root test (URT) using the augmented Dickey-Fuller (ADF) statistic (Wang et al., 2006; Özer and Ertokatl, 2010; Akintunde et al., 2015) as shown in Table 2. This result indicates that the levels of statistics are greater than critical value it follows that the data are stationary.

3.2. Analysis of R/S results

In this section, first we intended intricacy of ozone concentration using the rescaled range analysis. The Hurst exponent was estimated by AR (1) residuals of original total monthly ozone and seasonal time series data to remove linear dependence (Weng et al., 2008). The procedure of rescaled range analysis of different residual samples of ozone concentrations in the study period are given in Table 3. The slope of the best fitted line computed by log-log of '(R/S)' vs. 'N' using procedure in Fig. 2 gives the estimation of the Hurst exponent (H = 0.2622) as indicated in Fig. 11 with 87.6% significant at 95% confidence interval and obtained the fractal dimension value using equation (7) is D = 1.7582. This revealed that as the value of H is less than 0.5 and fractal value D > 1.5, the ozone concentration has shown anti-persistence behavior i.e. negative correlation with all lags.

The Fig. 12 shows the results of Hurst exponent obtained for each of the months from January to December in the study region from 1970 to 2013. The results for fractal value related to Hurst exponent for each month revealed in Table 4. The fractal behavior of the seasonal pattern of ozone indicates anti-persistence (i.e., negative correlation) of time series for all months except November which shows persistence i.e. time series

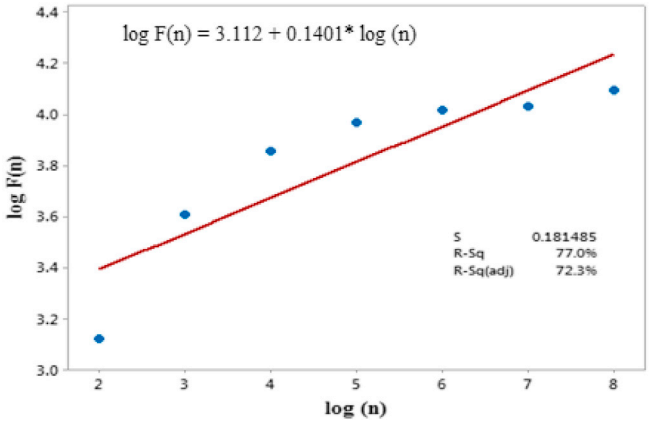

Fig. 13. Plot of Detrended fluctuation analysis of the ozone concentration data over the Pakistan atmospheric region, where the slope of fitted line indicates scaling exponent for the year from 1970-2013.

Table 6
Estimation of log (n) and log *F(n)* of Detrended fluctuation of original time series for the ozone concentration over the Pakistan atmospheric region from 1970-2013.

Level	(n)	F(n)	$\log_2(n)$	$\log_2 F(n)$
1	256	17.068	8	4.0932
2	128	16.332	7	4.0296
3	64	16.193	6	4.0173
4	32	15.634	5	3.9666
5	16	14.462	4	3.8542
6	8	12.207	3	3.6096
7	4	8.692	2	3.1197

Table 5
Summary of Detrended fluctuation analysis for ozone series.

H estimate							0.1401
Domain							Time
Statistic							RMSE
Length of series							528
Block detrending model							X ~ 1 + t
Block overlap fraction							0
Scale ratio							2
Scale:	4.000	8.000	16.000	32.000	64.000	128.000	256.000
RMSE:	8.692	12.207	14.462	15.634	16.193	16.332	17.068

is independent, increasing trend and positive correlation of the time series.

3.3. Analysis of DFA results

Albeit, the objective of DFA is to evaluate the correlation properties of the stratospheric ozone data over the Pakistan atmospheric region as presented in summary Table 5. These results estimated using equations (5) and (6). The value of scaling exponent 'α' is less than 0.5 ($\alpha = 0.1401$) as revealed in Fig. 13. Therefore, the time series of ozone data is anti-correlation and the significance of lest square procedure is only 77.0% with 95% confidence interval. The result of estimation for Detrended fluctuation analysis by log-log process revealed in Table 6.

3.4. Tests for dependence of nonlinearity

Moreover, many researchers used BDS statistic to observe the evidence of the nonlinearity of time series data (Weng et al., 2008; Özer and Ertokatl, 2010; Halkos and Tsilika, 2014) of AR (1) residuals which is non-parametric test. In our case, we have chosen a parametric test the Anderson-Darling test statistic that compares whether the data is linear or nonlinear. For this purpose, the null hypothesis of linearity of data considered as Ho: Linear and Ha: Non-linear. For ozone time series data it was considered as, the critical value of $A^2 = 0.787$, Ho: Linear: $A^2 <$ 0.787, Ha: Non-linear" $A^2 > 0.787$ (Stephens, 1974). From Fig. 8 the value of AD-statistics is 3.729 (with associated *p*-value less than 0.005) that is greater than the critical value. Therefore, we reject the null hypothesis and conclude that the ozone concentration is non-linearly dependent, because linear dependence has been removed.

4. Conclusion

In order to examine the nonlinear variations of the stratospheric ozone level at Pakistan atmospheric region, basic statistic analysis and fractal dimension analysis associated with the Hurst exponent methodology was performed using rescaled range analysis. The monthly data of stratospheric ozone from 1970 to 2013 are used and seasonal data extracted from the original data. Basic statistical analysis described the fluctuation of ozone thickness and intrinsic behavior and also linear robust interdependence clearly exist (Weng et al., 2008) in time series data of ozone in Pakistan region. The evidence found here has confirmed the best fitted AR (1) model that reduces the linearity from ozone series data.

Further, this communication might include fractal analysis to study the dynamics of ozone level. The results obtained have shown that all monthly and seasonal time series data on stratospheric ozone concentration have anti-persistent behavior (negative correlation) except the month of November that shown persistence i.e. positive correlation in time series. The Hurst exponent results obtained from residuals of ozone time series are a valuable numerical estimation that can used to illustrate the series behavior of the randomness of ozone in a different atmosphere.

Nevertheless, we also carried out a comparative analysis of the performance of both DFA and R/S analysis methods to identify the fractal character of ozone concentration. Our results have emphasized that both methods shown similar results as an ant-persistent (an anti-correlation) and R/S analysis is more appropriate than DFA to identify the fractal nature of time series data. Since, our data follows stationarity and R/S analysis is applied always for stationary time series data while DFA is applicable for non-stationary time series (Matsoukas et al., 2000).

Finally, the test statistics also confirmed the nonlinear behavior of ozone level at Pakistan atmospheric region. The hypothesis of linearity was rejected by the AD statistics test. Indeed, the results permitted rejecting the null hypothesis of the monthly ozone level being non-linear dependence and the evidences indicate that the non-linear intrinsic structure was present in the ozone data, The basic objective of this research was to identify the fractal nature in the stratospheric region of Pakistan. In future, this study may be extended to the investigation of the natural complexity of the stratospheric ozone by correlation dimension and multi-fractal techniques and also by Wavelet transform.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.jastp.2018.01.013.

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